

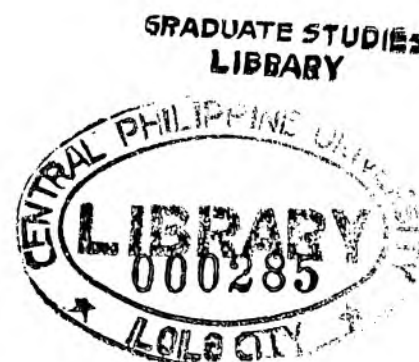
ON APPLICATIONS OF THE RETRACING METHOD FOR  
DISTANCE-REGULAR GRAPH

A Thesis  
Presented to  
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# Abstract

This thesis is an exposition of the article written by Akira Hiraki entitled Applications of Retracing Method for Distance-Regular Graphs published in European Journal of Combinatorics, April 2004. The main results of the article are as follows:

**Theorem 1.1** Let  $\Gamma$  be a distance-regular graph of diameter  $d$  with

$$r = |\{ i | (c_i, a_i, b_i) = (c_1, a_1, b_1) \}| \geq 2$$

and  $c_{r+1} \geq 2$ . Let  $m, s$  and  $t$  be positive integers with  $s \leq m, m + t \leq d$  and  $(s, t) \neq (1, 1)$ . Suppose  $b_{m-s+1} = \cdots = b_m = 1 + b_{m+1}$ ,  $c_{m+1} = \cdots = c_{m+t} = 1 + c_m$  and  $a_{m-s+2} = \cdots = a_{m+t-1} = 0$ . Then the following hold.

- (1) If  $b_{m+1} \geq 2$ , then  $t \leq r - 2 \lfloor s/3 \rfloor$ .
- (2) If  $c_m \geq 2$ , then  $s \leq r - 2 \lfloor t/3 \rfloor$ .

**Corollary 1.2.** Under the assumption of Theorem 1.1, the following hold.

- (1) If  $r = t$  and  $b_{m+1} \geq 2$ , then  $s \leq 2$ .
- (2) If  $r = s$  and  $c_m \geq 2$ , then  $t \leq 2$ .

**Corollary 1.3.** Let  $\Gamma$  be a distance-regular graph of valency  $k \geq 3$  with

$c_1 = \cdots = c_r = 1$ ,  $c_{r+1} = \cdots = c_{r+t} = 2$  and  $a_1 = \cdots = a_{r+t-1} = 0$ .

(1) If  $k \geq 4$ , then  $t \leq r - 2 \lfloor r/3 \rfloor$ .

(2) If  $2 \leq t = r$ , then  $\Gamma$  is either the Odd graph, or the doubled Odd graph.

(3) If  $2 \leq t = r - 1$ , then  $\Gamma$  is the Foster graph.